Spring 2016 Math 245 Mini Midterm 2 Solutions

1. Consider $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = x\lfloor x \rfloor$. Prove or disprove that f is injective.

False. We have $f(0) = 0\lfloor 0 \rfloor = 0 = \frac{1}{2}\lfloor \frac{1}{2} \rfloor = f(\frac{1}{2})$, but $0 \neq \frac{1}{2}$.

2. Let A, B, C be sets, with $B \subseteq C$. Prove that $(A \times B) \subseteq (A \times C)$.

Let $x \in A \times B$ be arbitrary. There must be some $a \in A, b \in B$ such that $x = (a, b)$. Since $B \subseteq C$, in fact $b \in C$. Hence $x = (a, b) \in A \times C$. Therefore $(A \times B) \subseteq (A \times C)$.

3. Carefully define each of the following terms:

a. relation

A *relation* from set A to set B is a subset of $A \times B$.

b. symmetric (relation)

A relation R is *symmetric* if whenever $(a, b) \in R$, we must have $(b, a) \in R$.

c. equivalence relation

A relation is an *equivalence relation* if it is reflexive, symmetric, and transitive.

d. partial order

A relation is a *partial order* if it is reflexive, antisymmetric, and transitive.

e. surjective

A function $f : A \to B$ is *surjective* if for every $b \in B$ there is at least one $a \in A$ such that $f(a) = b$.

4. Consider the relation R on $\mathbb{Z}$ given by $aRb \Leftrightarrow |a - b| \leq 1$. Prove or disprove that R is transitive.

False. We have $3R2$ since $|3 - 2| \leq 1$. We have $2R1$ since $|2 - 1| \leq 1$. But $3 \not R 1$ since $|3 - 1| > 1$.

5. Find the general solution to the recurrence relation $a_n = -a_{n-1} + 6a_{n-2}$.

This relation has characteristic equation $r^2 = -r + 6$, which rearranges as $r^2 + r - 6 = 0$, and factors as $(r+3)(r-2) = 0$. There are two roots, -3 and 2, so the general solution is $a_n = A(-3)^n + B(2)^n$.